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MODELLING THE TRANSMISSION DYNAMICS OF LASSA HEMORRHAGIC FEVER WITH MITIGATING MEASURES USING FRACTIONAL ORDER DERIVATIVES

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ABSTRACT

This study proposes a fractional-order model, formulated in the sense of the Caputo fractional derivative, to investigate the transmission dynamics of Lassa hemorrhagic fever. A necessary qualitative analysis was conducted to assess the validity of the model. The fundamental number of reproduction (R_0), was estimated using the next generation matrix approach and was found to be less than unity (1) through the stability analysis. The Jacobi and Lyapunov approach was utilized to construct the stability analysis at Infection free and Endemic Equilibrium respectively. The fractional order model is analyzed using a modified Adams-Bashforth estimator-corrector technique. Additionally, the numerical simulations were done to validate the effect of various parameters on the model as a whole. The graphical solutions indicate that the fractional order ß, which is the saturation factor impact the dynamics of the model when varied. The results indicate that saturation of infectious individuals in the system helps flatten the infection transmission curve thereby achieving a disease free community in the long run.

1 INTRODUCTION

The deadly Lassa virus is an animal-borne infectious disease that is transported and spread by the African rat, commonly referred to as the "multimammate natal rat" [1, 2]. The infectious agent is a virus made up of one strand of RNA which belongs to a class of Arenaviridae viral group, which is typically linked to human disease that is spread by rodents. In certain parts of the world, infections with the arena virus are very prevalent in people and can result in serious diseases. The mean size of the round virus particles ranges between 110 and 130 micromillimetre. All are covered in a lipid membrane, and when examined broadly, they display gritty granules which are cellular, taken up from the cells that they inhabit [3]. In 1969, Lassa fever (LF) was first identified in the town of Lassa, located in the Yedseram River area in what is now Borno State in northern Nigeria [4]. The initial casualty was Wine Laura, a 65-year-old caregiver who served at Lassa Clinic Mission. Since Lassa fever's original discovery in Lassa, Nigeria, epidemics have occurred frequently within the rural and non-urban areas of Nigerian, including Ekpoma, Onitsha, Jos, Zorikwa, and so on. In Liberia, Sierra Leone Guinea, Nigeria and other West African nations, Lassa fever is prevalent and affects 2-3 million individuals yearly, killing 6000–12,000 people [5, 6]. The West African nations of Liberia and other neighbouring countries have experienced epidemics, according to the ProMED International Society for Infectious Diseases.

Although no major epidemics have ever been reported in Ghana, Cote d'Ivoire, Benin, or Togo, isolated cases indicate the presence of viral infections in these countries. Nevertheless, certain strains of Lassa fever that originated elsewhere are being brought into the United Kingdom and United States by the tourists [6-8]. (LF) is prevalent in Nigeria, with the yearly rise of human cases often occurring in the times of drought after the Mastomys rats' reproductive cycle during the rainy season. Nonetheless, because (LHF) is persistent in Nigeria, the nation has developed the ability to control outbreaks; as a result, the present national risk level for the disease is regarded as moderate [9-10]. Mastomys rodents are occasionally eaten as food, and infection can occur after they are captured and processed. Inhaling microscopic airborne particles tainted with infected rodents excreta can also result in infection with the (LHF) virus. Cleaning and other

maintenance activities may release aerosols or cause transmission through the air. Person-to-person transmission of Lassa infection can also occur through close contact with infectious rodents, and the virus may further spread through contact with the secretions, tissues, blood, or excreta of infected humans (CDC) [10]. Depending on socioeconomic status, Onah et al. developed an adaptive structure for the Lassa virus model in 2020. Research on the disease's impact on the economy was conducted globally [11]. Peter et al. updated a basic disease model for Lassa hemorrhagic fever with the optimal control strategies in 2020 [12]. By adding the seasonal factors, Bakare et al.'s 2020 study on the disease's transmission patterns resulted in a nonlinear ordinary differential equation. Stochastic approaches are useful in data analysis, particularly in mathematical modeling contexts, for quantitative research [13, 14, 15]. Epidemiology [16], energy [17], and Finance [18]. Certain actions including community cleanliness, outreach efforts, preventative measures, and the isolation of affected individuals as a form of control against Lassa fever are advised for flattening the curves for disease propagation, [19]. Collins et al. constructed an analytical system for Lassa virus management methods in 2021. The system was stratified into higher and lower socioeconomic groups according to epidemiological criteria, and prevention strategies such as therapies, public awareness campaigns, community sanitation, and rodent control were examined [20].

Many researchers have used fractional-order models to describe the dynamics of various infectious and non-infectious diseases in the modern era [21-31]. Analytical solution of a mathematical framework using fractional order for tumor displaying mutations in cells was investigated in [32-34], [26]. Investigation of optimal control in co-infection scenarios and bifurcation of SARS-CoV-2 and ZIKA was done in [35-37]. Fractional order approaches were used to investigate the dynamics of complex systems in [38-40]. The studies in [27–30, 39–43] provide fundamental principles for analysing complex systems and their modelling, which collectively facilitate the development of clear methodological frameworks in this field. Atede et al. [37] focused on Nigerian actual data as a base of study and investigated a fractional model incorporating vaccination as a control strategy for COVID-19 that took ecological propagation into consideration. Studies of TB in

relation to internal reactivated and foreign re-infection have been conducted using Frégal-order models [39-40].

1.1 Objectives

This study examines the dynamics and management of Lassa hemorrhagic fever (LHF) transmission within the population using a fractional-order modeling framework. It also attempts to show how the fractional order variable and memory indices influence the dynamics of the disease.

1.2 Structure of the Article

Section 2 presents established results required for the study. In Section 3, we construct a fractional-order model for Lassa hemorrhagic fever (LHF) based on the Caputo fractional derivative. Section 4 includes some basic results for the model using fixed-point. The analysis of the model stability was verified in section 5. In Section 6, the Adams bashforth estimator-corrector method was used for the approximate solution of the model. Section 7 discourse the hereditary (congenital) trace and memory tracking. Section 8 discuss the numerical simulations using MATLAB software. Finally, the study's conclusion was discussed in Section 9

1.3 Preliminaries and Definitions

We include some notations, definitions, and established results that are required for the sequel. This study uses Louiville-Caputo's fractional derivative

Definition 1: let $v \in U^1(x,y), \quad y > x, 0 < \beta \leq 1$, the operator of the Caputo derivative is denoted as [26-31, 44-45];

$$ {}_0^c D_t^{\beta} v(t) = \frac{N(\beta)}{(1-\beta)} \int_x^t \exp\left[\frac{-\beta(t-\tau)}{1-\beta}\right] v(\tau) d\tau, \quad t \geq 0 \tag{1}$$

Where the function of Normalization is N which relies on ß and yields $N(0) = N(1) = 1$

${}_0^c D_t^{\beta} v(t) = 0$, if v is a constant function, (1) can also be alternatively given as;

$$ {}_0^c D_t^{\beta} v(t) = \frac{\beta N(\beta)}{1-\beta} \int_x^t (v(t) - v(\tau)) \exp\left[\frac{\beta(t-\tau)}{1-\beta} \right] d\tau $$

The above equation holds if the function is not included in $U^1(x, y)$

Definition 2: let $0 < \beta \leq 1$, the operator of the Caputo integral of order β is denoted as [27, 44 - 45];

$$ {}_0^c I_t^{\beta} v(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)} v(t) + \frac{2\beta}{(2-\beta)N(\beta)} \int_{t_0}^t v(\tau) d\tau, \quad t \geq 0 \qquad \textbf{(2)} $$

2 MATERIAL AND METHODS

2.1 Formulation of Model.

Fractional order model is used in mathematics to represent the elaborate behaviors of disease structures, providing a more practical representation than integer models. This technique expands the capacity of integer-order models and has proven to be an indispensable tool for infectious disease modeling. These models incorporate important features such as past disease status, memory of previous disease patterns (for fractional orders less than 1), and genetic variability. Furthermore, unlike ordinary differential equations, whose dynamics are typically local in nature, fractional-order derivatives exhibit a global (nonlocal) character. The efficiency of fractional calculus advances as the uniformity of range of the system gets better [26-31, 46]

Moving forward, to illustrate the dynamics of Lassa hemorrhagic fever (LHF) transmission across rodent and human populations, we construct a fractional mathematical model for the disease. The model makes some assumptions; three rodent compartments and four human compartments make up our seven compartmental models which is equipped with biological parameters. The total rodent population is divided into three classes which are (S_r) = susceptible, Exposed =(E_r) and Infected= (I_r). The human population as a whole is split into four classes: Susceptible = (S_h), Exposed= (E_h) Infected= (I_h) and Recovered = (R_h). This compartmental structure is represented in the schematic flow diagram below in Figure 1.

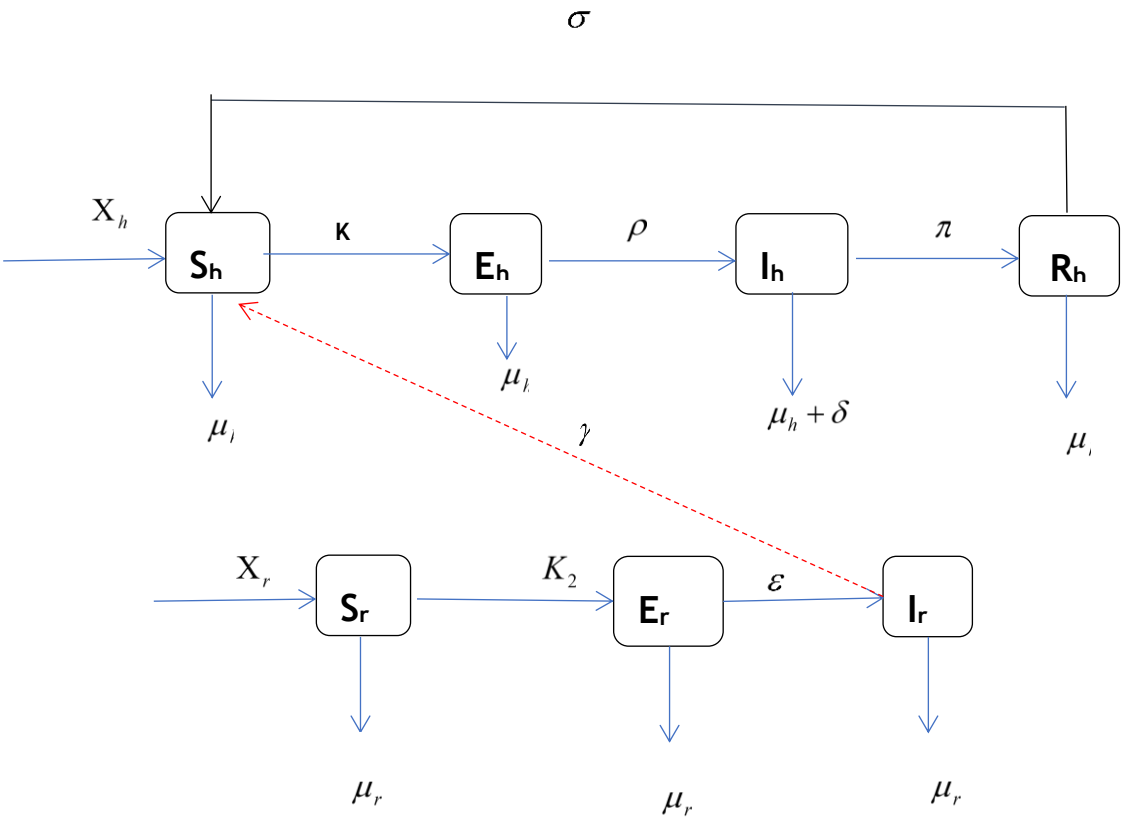


***Figure 1**. Schematic flow of proposed fractional model.*

The set of fractional differential equations (3) are derived from Figure 1

$$\left.\begin{aligned}
{}_0^cD_t^\beta S_h(t) &= X_h - (\kappa_1 + \mu_h)S_h + \sigma R_h + \gamma I_r \\
{}_0^cD_t^\beta E_h(t) &= \kappa_1 S_h - (\rho + \mu_h)E_h \\
{}_0^cD_t^\beta I_h(t) &= \rho E_h - I_h(\pi + \mu_h + \delta) \\
{}_0^cD_t^\beta R_h(t) &= \pi I_h - (\mu_h + \sigma)R_h \\
{}_0^cD_t^\beta S_r(t) &= X_r - (\kappa_2 + \mu_r)S_r \\
{}_0^cD_t^\beta E_r(t) &= \kappa_2 S_r - (\mu_r + \varepsilon)E_r \\
{}_0^cD_t^\beta I_r(t) &= \varepsilon E_r - (\mu_r + \gamma)I_r
\end{aligned}\right\} \quad \textbf{(3)}$$

Here ${}_0^cD_t^\beta(.)$ for $0 < \beta \leq 1$ in the scope of Liouville-Caputo Fractional of order β

$$\kappa_1 = \frac{e_h I}{1 + a_h I}, \quad \kappa_2 = \frac{e_r I}{1 + a_r I}$$

Where e_r and e_h is the effective contact rate for rodent and human and population respectively a_h and a_r are the saturation factor for human and rodents population respectively.

***Table 1**. Parameters and their respective description.*

Parameters	Descriptions (Biological Meaning)
X_h	Human recruitment rate
κ_1	Incidence rate for human
κ_2	Incidence rate for rodent
σ	Relapse rate
ρ	Progression rate to infectious class
π	Rate of Progression to recovered
X_r	rodent recruitment rate
ε	Progression rate to infected rodent
δ	Rate of disease induced death
μ_r	rate of natural death for rodent
μ_h	death rate for human
γ	Transmission rate from I_r to S_h

2.2 Existence and Uniqueness Solution of the Fractional Order Model (3)

Here we establish the existence of the FOM (3) solution by utilizing the Fixed-Point approach

Let $\Phi = (M(L))^7$ and $M(L)$ be a Banach Space which is continuous on $L \subset \mathbb{R} \to \mathbb{R}$ valued function, equipped with the norm

$\|S_h, E_h, I_h, R_h, S_r, E_r, I_r\| = \|S_r\| + \|E_r\| + \|I_r\| + \|S_h\| + \|E_h\| + \|I_r\| + \|R_h\|$, here $(\|.\|)$ connotes the supremum norm in $M(L)$ [47-48], applying the integral operator in definition (2) [30] on FOM (3) we obtain;

$$S_h(t) - S_h(0) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}[X_h - (\mu_h + \kappa_1)S_h(t) + \sigma R_h(t) + \gamma I_r(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[X_h-(\kappa_1+\mu_h)S_h(\tau)+\sigma R_h(\tau)+\gamma I_r(\tau)]d\tau$$

$$E_h(t)-E_h(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\kappa_1 S_h(t)-(\rho+\mu_h)E_h(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\kappa_1 S_h(\tau)-(\rho+\mu_h)E_h(\tau)]d\tau$$

$$I_h(t)-I_h(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\rho E_h(t)-(\mu_h+\delta+\pi)I_h(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\rho E_h(\tau)-(\mu_h+\delta+\pi)I_h(\tau)]d\tau$$

$$R_h(t)-R_h(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\pi I_h(t)-(\mu_h+\sigma)R_h(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\pi I_h(\tau)-(\mu_h+\sigma)R_h(\tau)]d\tau$$

$$S_r(t)-S_r(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[X_r-(\kappa_2+\mu_r)S_r(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[X_r-(\kappa_2+\mu_r)S_r(\tau)]d\tau$$

$$E_r(t)-E_r(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\kappa_2 S_r(t)-(\mu_r+\varepsilon)E_r(t)]$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\kappa_2 S_r(\tau)-(\mu_r+\varepsilon)E_r(\tau)]d\tau$$

$$I_r(t)-I_r(0)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\varepsilon E_r(t)-(\mu_r+\gamma)I_r(t)]+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\varepsilon E_r(\tau)-(\mu_r+\gamma)I_r(\tau)]d\tau$$

Alternatively, the above equations can be given as;

$$\left.\begin{aligned}
\Gamma_1(S_h,t) &= X_h-(\kappa_1+\mu_h)S_h(t)+\sigma R_h(t)+\gamma I_r(t)\\
\Gamma_2(E_h,t) &= \kappa_1 S_h(t)-(\rho+\mu_h)E_h(t)\\
\Gamma_3(I_h,t) &= \rho E_h(t)-(\mu_h+\delta+\pi)I_h(t)\\
\Gamma_4(R_h,t) &= \pi I_h(t)-(\mu_h+\sigma)R_h(t)\\
\Gamma_5(S_r,t) &= X_r-(\kappa_2+\mu_r)S_r(t)\\
\Gamma_6(E_r,t) &= \kappa_2 S_r(t)-(\mu_r+\varepsilon)E_r(t)\\
\Gamma_7(I_r,t) &= \varepsilon E_r(t)-(\mu_r+\gamma)I_r(t)
\end{aligned}\right\} \quad \textbf{(4)}$$

To verify the below theorem let $k_1 \ldots k_7$, be positive constant greater than 0 that is $(k_1 \ldots k_7 > 0)$ such that

$$\|S_h\| \le k_1, \|E_h\| \le k_2, \|I_h\| \le k_3, \|R_h\| \le k_4, \|S_r\| \le k_5, \|E_r\| \le k_6, \|I_r\| \le k_7$$

Let's denote $c_1 = \kappa_1 + \mu_h$, $c_2 = \mu_h + \rho$, $c_3 = \mu_h + \pi + \delta$, $c_4 = \sigma + \mu_h$, $c_5 = \kappa_2 + \mu_r$, $c_6 = \mu_r + \varepsilon$ and $c_7 = \mu_r + \gamma$

Theorem 1: The Lipchitz contraction and condition is satisfied if the kernels $\Gamma_1 \ldots \Gamma_7$ hold true for the inequality

$$0 \le c_j < 1 \text{ where } j = 1 \ldots 7 \quad \textbf{(5)}$$

Proof:

Assume that $S_{h,1}$ and $S_{h,2}$ are functions such that;

$$\begin{aligned}
\|\Gamma_1(S_{h,1},t)-\Gamma_1(S_{h,2},t)\| &= \|(-\kappa_1-\mu_h)(S_{h,1}-S_{h,2})\| \le [\|\kappa_1+\mu_h\|\|S_{h,1}-S_{h,2}\|]\\
&\le (\kappa_1+\mu_h)\|S_{h,1}(t)-S_{h,2}(t)\|\\
&\le c_1\|S_{h,1}-S_{h,2}\|
\end{aligned} \quad \textbf{(6)}$$

Therefore

$$\|\Gamma_1(S_{h,1},t)-\Gamma_1(S_{h,2},t)\| \le c_1\|S_{h,1}-S_{h,2}\| \quad \textbf{(7)}$$

Thus for Γ_1 the condition of Lipschitz is satisfied

In the same manner $\Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6, \Gamma_7$ lipschitz's condition can be easily established and obtained as follows;

$$\begin{aligned}
&\left\|\Gamma_2(E_{h,1},t)-\Gamma_2(E_{h,2},t)\right\| \le c_2\left\|E_{h,1}-E_{h,2}\right\| \\
&\left\|\Gamma_3(I_{h,1},t)-\Gamma_3(1_{h,2},t)\right\| \le c_3\left\|I_{h,1}-I_{h,2}\right\| \\
&\left\|\Gamma_4(R_{h,1},t)-\Gamma_4(R_{h,2},t)\right\| \le c_4\left\|R_{h,1}-R_{h,2}\right\| \\
&\left\|\Gamma_5(S_{r,1},t)-\Gamma_5(S_{r,2},t)\right\| \le c_5\left\|S_{r,1}-S_{r,2}\right\| \\
&\left\|\Gamma_6(E_{r,1},t)-\Gamma_6(E_{r,2},t)\right\| \le c_6\left\|E_{r,1}-E_{r,2}\right\| \\
&\left\|\Gamma_7(I_{r,1},t)-\Gamma_7(I_{r,2},t)\right\| \le c_5\left\|I_{r,1}-I_{r,2}\right\|
\end{aligned} \tag{8}$$

The Fractional Order Model (3) can be given in a looping form using definition (2) and so obtain;

$$\left.\begin{aligned}
S_{h,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_1(S_{h,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_1(S_{h,i-1},\tau)d\tau \\
E_{h,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_2(E_{h,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_2(E_{h,i-1},\tau)d\tau \\
I_{h,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_3(I_{h,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_3(I_{h,i-1},\tau)d\tau \\
R_{h,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_4(R_{h,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_4(R_{h,i-1},\tau)d\tau \\
S_{r,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_5(S_{r,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_5(S_{r,i-1},\tau)d\tau \\
E_{r,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_6(E_{r,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_1(E_{r,i-1},\tau)d\tau \\
I_{r,i}(t) &= \frac{2(1-\beta)}{(2-\beta)N(\beta)}\Gamma_7(I_{r,i-1},t)+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\Gamma_7(I_{r,i-1},\tau)d\tau
\end{aligned}\right\}$$

The positive initial condition is given as;

$$S_h(t)\ge 0,\ E_h(t)\ge 0,\ I_h(t)\ge 0,\ R_h(t)\ge 0,\ S_r(t)\ge 0,\ E_r(t)\ge 0,\ I_r(t)\ge 0, \tag{9}$$

Hence the difference between the sequential terms is given by;

$$P_i^1 = S_{h,i}(t) - S_{h,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_1(t, S_{h,i-1}) - \Gamma_1(t, S_{h,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_1(\tau, S_{h,i-1}) - \Gamma_1(\tau, S_{h,i-2}))d\tau \quad (10)$$

$$P_i^2 = E_{h,i}(t) - E_{h,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_2(t, E_{h,i-1}) - \Gamma_2(t, E_{h,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_2(\tau, E_{h,i-1}) - \Gamma_1(\tau, E_{h,i-2}))\ d\tau$$

$$P_i^3 = I_{h,i}(t) - I_{h,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_3(t, I_{h,i-1}) - \Gamma_3(t, I_{h,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_3(\tau, I_{h,i-1}) - \Gamma_3(\tau, I_{h,i-2}))d\tau$$

$$P_i^4 = R_{h,i}(t) - R_{h,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_4(t, R_{h,i-1}) - \Gamma_4(t, R_{h,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_4(\tau, R_{h,i-1}) - \Gamma_4(\tau, R_{h,i-2}))d\tau$$

$$P_i^5 = S_{r,i}(t) - S_{r,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_5(t, S_{r,i-1}) - \Gamma_5(t, S_{r,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_5(\tau, S_{r,i-1}) - \Gamma_5(\tau, S_{r,i-2}))d\tau$$

$$P_i^6 = E_{r,i}(t) - E_{r,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_6(t, E_{r,i-1}) - \Gamma_6(t, E_{r,i-2})) + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_6(\tau, E_{r,i-1}) - \Gamma_6(\tau, E_{r,i-2}))d\tau$$

$$P_i^7 = I_{r,i}(t) - I_{r,i-1}(t) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}(\Gamma_7(t, I_{r,i-1}) - \Gamma_7(t, I_{r,i-2}))$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}(\Gamma_7(\tau,I_{r,i-1})-\Gamma_7(\tau,I_{r,i-2}))d\tau$$

Applying the Lipschitz condition obtained in (7) and using norm on both sides of (10), with the application of triangular inequality, we get;

$$\left\|P_i^1(t)\right\|=\left\|S_{h,i}(t)-S_{h,i-1}(t)\right\|\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1\left\|S_{h,i-1}-S_{h,i-2}\right\|$$

$$+\frac{2\beta}{(2-\beta)N(\beta)}c_1\int_{t_0}^{t}\left\|S_{h,i-1}-S_{h,i-2}\right\|d\tau$$

Which yields;

$$\left\|P_i^1(t)\right\|\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1\left\|P_{i-1}^1(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_1\int_{t_0}^{t}\left\|P_{i-1}^1(t)\right\|d\tau$$

In the same manner we have for the other set of equations;

$$\left.\begin{aligned}
\left\|P_i^2(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_2\left\|P_{i-1}^2(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_2\int_{t_0}^{t}\left\|P_{i-1}^2(\tau)\right\|d\tau\\
\left\|P_i^3(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_3\left\|P_{i-1}^3(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_3\int_{t_0}^{t}\left\|P_{i-1}^3(\tau)\right\|d\tau\\
\left\|P_i^4(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_4\left\|P_{i-1}^4(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_4\int_{t_0}^{t}\left\|P_{i-1}^4(\tau)\right\|d\tau\\
\left\|P_i^5(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_5\left\|P_{i-1}^5(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_5\int_{t_0}^{t}\left\|P_{i-1}^5(\tau)\right\|d\tau\\
\left\|P_i^6(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_6\left\|P_{i-1}^6(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_6\int_{t_0}^{t}\left\|P_{i-1}^6(\tau)\right\|d\tau\\
\left\|P_i^7(t)\right\|&\le\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_7\left\|P_{i-1}^7(t)\right\|+\frac{2\beta}{(2-\beta)N(\beta)}c_7\int_{t_0}^{t}\left\|P_{i-1}^7(\tau)\right\|d\tau
\end{aligned}\right\}\quad\textbf{(11)}$$

Subsequently due to the aforementioned expressions we obtain the below theorem.

Theorem 2: the FOM (3) solution exist and it is unique if t_0 exit such that t_0 is found in

$$\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_j + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_j < 1, \text{ where } j = 1...7 \quad \textbf{(12)}$$

Proof:

For the functions $S_r(t), E_r(t), I_r(t), S_h(t), E_h(t), I_h(t)$ and $R_h(t)$ remains within limit (bounded) and satisfies the Lipchitz criterion by considering (11) we obtain;

$$\left\|P_i^1(t)\right\| \le \left\|S_{h,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_1\right]^i,$$

$$\left\|P_i^2(t)\right\| \le \left\|E_{h,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_2 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_2\right]^i,$$

$$\left\|P_i^3(t)\right\| \le \left\|I_{h,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_3 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_3\right]^i,$$

$$\left\|P_i^4(t)\right\| \le \left\|R_{h,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_4 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_4\right]^i,$$

$$\left\|P_i^5(t)\right\| \le \left\|S_{r,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_5 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_5\right]^i,$$

$$\left\|P_i^6(t)\right\| \le \left\|E_{r,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_6 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_6\right]^i,$$

$$\left\|P_i^7(t)\right\| \le \left\|I_{h,i}(0)\right\| \left[\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_7 + \frac{2\beta t_0}{(2-\beta)N(\beta)}c_7\right]^i$$

Hence we have been able to establish the continuity and existence and of the FOM (3).

To verify that the above expressions satisfies FOM (3), let;

$$\left.\begin{aligned} S_h(t) &= S_h(0) + S_{h,i}(t) - Q_i^1(t) \\ E_h(t) &= E_h(0) + E_{h,i}(t) - Q_i^2(t) \\ I_h(t) &= I_h(0) + I_{h,i}(t) - Q_i^3(t) \\ R_h(t) &= R_h(0) + R_{h,i}(t) - Q_i^4(t) \\ S_r(t) &= S_r(0) + S_{r,i}(t) - Q_i^5(t) \\ E_r(t) &= E_r(0) + E_{r,i}(t) - Q_i^6(t) \\ I_r(t) &= I_r(0) + I_{r,i}(t) - Q_i^7(t) \end{aligned}\right\} \quad \textbf{(13)}$$

Applying the norm operator on 13 gives;

$$\left\|Q_i^1(t)\right\| \le \frac{2(1-\beta)}{(2-\beta)N(\beta)}\left\|\Gamma_1(S_h,t) - \Gamma_1(S_{h,i-1},t)\right\| + \frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\left\|\Gamma_1(S_h,\tau) - \Gamma_1(S_{h,i-1},\tau)\right\|d\tau \quad \textbf{(14)}$$

$$\le \frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1\left\|S_h - S_{h,i-1}\right\| + \frac{2\beta t}{(2-\beta)N(\beta)} + c_1\left\|S_h - S_{h,i-1}\right\|$$

Looping the process gives;

$\left\|Q_i^1(t)\right\| \le \left(\frac{2(1-\beta)}{(2-\beta)N(\beta)} + \frac{2\beta t}{(2-\beta)N(\beta)}\right)^{i+1} c_1^{i+1}u_1$ Where $u_1 > 0$ is a constant, at t_0 giving;

$$\left\|Q_i^1(t)\right\| \le \left(\frac{2(1-\beta)}{(2-\beta)N(\beta)} + \frac{2\beta t_0}{(2-\beta)N(\beta)}\right)^{i+1} c_1^{i+1}u_1$$

$i \to \infty$ gives $\left\|Q_i^1(t)\right\| \to 0$.

Approaching the rest of the state variables in the same manner we obtain;

$$\left\|Q_i^j(t)\right\| \to 0 \text{ where } j = 2,3,4...7$$

Therefore the existence proof has been verified.

Moving forward to establish the uniqueness of the FOM (3), we assume a solution set $S_{r,1}(t), E_{r,1}(t), I_{r,1}(t), S_{h,1}(t), E_{h,1}(t), I_{h,1}(t)$ and $R_{h,1}(t)$

Say;

$$S_h(t)-S_{h,1}(t)=\frac{2(1-\beta)}{(2-\beta)N(\beta)}[\Gamma_1(S_h,t)-\Gamma_1(S_{h,1},t)]+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}[\Gamma_1(S_h,\tau)-\Gamma_1(S_{h,1},\tau)]d\tau \quad \textbf{(15)}$$

With the application of norm $(\|.\|)$ operator on (15) gives;

$$\begin{gathered}\|S_h(t)-S_{h,1}(t)\|\leq \\ \frac{2(1-\beta)}{(2-\beta)N(\beta)}\|\Gamma_1(S_h,t)-\Gamma_1(S_{h,1},t)\|+\frac{2\beta}{(2-\beta)N(\beta)}\int_{t_0}^{t}\|\Gamma_1(S_h,\tau)-\Gamma_1(S_{h,1},\tau)\|d\tau\end{gathered} \quad \textbf{(16)}$$

Applying the Kernel's Lipschitz condition to (16) we get;

$$\|S_h(t)-S_{h,1}(t)\|\leq\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1\|S_h-S_{h,1}\|+\frac{2\beta}{(2-\beta)N(\beta)}c_1t\|S_h-S_{h,1}\|$$

Which yields;

$$\|S_h(t)-S_{h,1}(t)\|\left(1-\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1-\frac{2\beta}{(2-\beta)N(\beta)}c_1t\right)\leq 0$$

Obviously, $S_h(t)=S_{h,1}(t)$ if and only if

$$(1-\frac{2(1-\beta)}{(2-\beta)N(\beta)}c_1-\frac{2\beta}{(2-\beta)N(\beta)}c_1t)>0 \quad \textbf{(17)}$$

Using the same approach as above gives the following result;

$E_h(t)=E_{h,1}(t)$, $I_h(t)=I_{h,1}(t)$, $R_h(t)=R_{h,1}(t)$, $S_r(t)=S_{r,1}(t)$, $E_r(t)=E_{r,1}(t)$ and $I_h(t)=I_{h,1}(t)$

Therefore, the FOM (3) has a solution which is unique, since the condition in (12) is satisfied.

2.3 Boundedness and Non-Negativity of the Fractional Model (3)

In order to establish the non-negativity of solution, we assume that the parameters (Table 1) of the fractional model remain positive

$$\begin{gathered}{}_0^cD_t^{\beta}S_h(t)\big|_{S_h(0)=0}=X_h\geq 0 \\ {}_0^cD_t^{\beta}E_h(t)\big|_{E_h(0)=0}=\kappa_1S_h\geq 0\end{gathered} \quad \textbf{(18)}$$

$$ {}_0^cD_t^{\beta} I_h(t)\Big|_{I_h(0)=0} = \rho E_h \geq 0 $$

$$ {}_0^cD_t^{\beta} R_h(t)\Big|_{R_h(0)=0} = \pi I_h \geq 0 $$

$$ {}_0^cD_t^{\beta} S_r(t)\Big|_{S_r(0)=0} = X_r \geq 0 $$

$$ {}_0^cD_t^{\beta} E_r(t)\Big|_{E_r(0)=0} = \kappa_2 S_r \geq 0 $$

$$ {}_0^cD_t^{\beta} I_r(t)\Big|_{I_r(0)=0} = \varepsilon E_r \geq 0 $$

From (18) the solution sets of the model remain non-negative (positive)

Theorem 3: for $t > 0$ the fractional model (3) remains within limit and bounded

Proof

Let $N_h(t) = [S_h(t) + E_h(t) + I_h(t) + R_h(t)]$

So;

$$ {}_0^cD_t^{\beta} N_h(t) = X_h - (S_h + E_h + I_h + R_h)\mu_h $$

$$ {}_0^cD_t^{\beta} N_h(t) = X_h - \mu_h N_h(t) \qquad \textbf{(19)} $$

Using integrating factor method on (19) gives

$$ N_h(t) \leq \frac{X_h}{\mu_h} + Ae^{-\mu_h t} $$

where A denotes the constant of integration, as $t \to \infty$

$N_h(t) \leq \frac{X_h}{\mu_h}$, therefore, the viable region of the fractional model (3) for human class is $\Pi_h = [(S_h(t), E_h(t), I_h(t), R_h(t)) \in R_+^4 : N_h(t) \leq \frac{X_h}{\mu_h}]$

In the same manner we compute for vector (rodent) population

$$ \Pi_m = [(S_m(t), E_m(t), I_m(t) \in R_+^3 : 0 \leq N_m(t) \leq \frac{X_m}{\mu_m}] $$

Say;

$[(S_m(t) + E_m(t) + I_m(t) + S_h(t) + E_h(t) + I_h(t) + R_h(t)]$ Are positive and remain within limit (bounded) then there exist constants positive $L_1 \ldots L_7$ such that

$$\|S_h(t)\| \le L_1, \|E_h(t)\| \le L_2, \|I_h(t)\| \le L_3 \|R_h(t)\| \le L_4, \|S_r(t)\| \le L_5, \|E_r(t)\| \le L_6, \|I_r(t)\| \le L_7$$

Hence the solution sets of FOM (3) is bounded.

2.4 Points of Equilibrium

Equilibrium point is the steady state solution of fractional model (3) and two types of equilibrium points – equilibrium in the absence of infection and endemic equilibrium- will be discussed.

$${}_0^cD_t^\beta S_r(t) = {}_0^cD_t^\beta E_r(t) = {}_0^cD_t^\beta I_r(t) = {}_0^cD_t^\beta S_h(t) = {}_0^cD_t^\beta E_h(t) = {}_0^cD_t^\beta I_h(t) = {}_0^cD_t^\beta R_h(t) = 0$$

$$\begin{aligned}
&X_h - (\kappa_1 + \mu_h)S_h + \sigma R_h + \gamma I_r = 0\\
&K_1 S_h - (\rho + \mu_h)E_h = 0\\
&\rho E_h - I_h(\pi + \mu_h + \delta) = 0\\
&\pi I_h - (\sigma + \mu_h)R_h = 0\\
&X_r - (\kappa_2 + \mu_r)S_r = 0\\
&K_2 S_r - (\mu_r + \varepsilon)E_r = 0\\
&\varepsilon E_r - (\mu_r + \gamma)I_r = 0
\end{aligned} \qquad \textbf{(20)}$$

At disease free equilibrium point

$$P^0 = (S_r^0, E_r^0, I_r^0 S_h^0, E_h^0, I_h^0, R_h^0) = (\frac{X_r}{\mu_r}, 0, 0, \frac{X_h}{\mu_h}, 0, 0, 0) \qquad \textbf{(21)}$$

At endemic equilibrium point

$$P^* = (S_r^*, E_r^*, I_r^*, S_h^*, E_h^*, I_h^*, R_h^*) = (\frac{X_r}{(\kappa_2 + \mu_r)}, \frac{\kappa_2 S_r^*}{(\mu_r + \varepsilon)}, \frac{\varepsilon E_r^*}{(\mu_r + \gamma)}, \frac{\sigma R_h^* - X_h}{(\kappa_1 + \mu_h)}, \frac{\kappa_1 S_h^*}{(\rho + \mu_h)}, \frac{\rho E_h^*}{(\mu_h + \delta + \pi)}, \frac{\pi I_h^*}{(\mu_h + \sigma)}) \qquad \textbf{(22)}$$

2.4.1 Fundamental reproduction number (R_0)

R_0 of the FOM (3) will be computed using next generation matrix technique [34]. R_0 for human and rodent compartment will be done differently, $R_{0h} = \Delta(M_h V_h^{-1})$, $R_{0r} = \Delta(M_r V_r^{-1})$ where M_h and M_r is the Matrix of new infection for human and rodent compartment respectively while, V_h and V_r is the matrix of secondary infection in the human rodent compartment respectively and Δ denote the maximum absolute value of the matrix.

$$M_h = \begin{bmatrix} 0 & e_h S \\ 0 & 0 \end{bmatrix}, \quad V_h = \begin{bmatrix} -(\mu_h + \rho) & 0 \\ \rho & -(\delta + \mu_h + \pi) \end{bmatrix}, \quad R_{0h} = \Delta(M_h V_h^{-1})$$

The spectral radius Δ of $M_h V_h^{-1}$ is

$$R_{0h} = \frac{e_h X_h \rho}{\mu_h(\delta + \mu_h + \pi)(\rho + \mu_h)} \tag{23}$$

$$M_r = \begin{bmatrix} 0 & e_r S \\ 0 & 0 \end{bmatrix}, \quad V_r = \begin{bmatrix} -(\varepsilon + \mu_r) & 0 \\ \varepsilon & -\mu_r \end{bmatrix}, \quad R_{0r} = \Delta(M_r V_r^{-1})$$

The spectral radius Δ of $M_h V_h^{-1}$ is

$$R_{0h} = \frac{e_r X_r \varepsilon}{\mu_r^2(\varepsilon + \mu_r)} \tag{24}$$

2.4.2 Stability Analysis

The stability Analysis will be established with respect to R_0 at infection free and at endemic equilibrium

Theorem 4: if $R_{0h} < 1$ and $R_{0r} < 1$ then the FOM (3) is asymptotically stable locally at infection free equilibrium.

Proof:

Let

$$\begin{aligned} H_1 &= X_h - (K_1 + \mu_h)S_h + \sigma R_h + \gamma I_r \\ H_2 &= \kappa_1 S_h - (\rho + \mu_h)E_h \\ H_3 &= \rho E_h - (\mu_h + \delta + \pi)I_h \\ H_4 &= \pi I_h - (\mu_h + \sigma)R_h \\ H_5 &= X_r - (K_2 + \mu_r)S_r \\ H_6 &= \kappa_2 S_r - (\mu_m + \varepsilon)E_r \\ H_7 &= \varepsilon E_r - (\mu_r + \gamma)I_r \end{aligned} \tag{25}$$

Using the technique of Jacobi we obtain;

$$J_0(S_h, E_h, I_h, R_h, S_r, E_r, I_r) \tag{26}$$

$$J_0 = \begin{bmatrix} -\mu_h & 0 & e_h S & \sigma & 0 & 0 & \gamma \\ K_1 & -(\rho+\mu_h) & e_h S & 0 & 0 & 0 & 0 \\ 0 & \rho & -(\mu_h+\pi+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & \pi & -(\sigma+\mu_h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu_r & 0 & e_r S \\ 0 & 0 & 0 & 0 & 0 & -(\mu_r+\varepsilon) & e_r S \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\mu_r+\gamma) \end{bmatrix} \tag{27}$$

Systemically solving (27) gives;

$\lambda_1 = -\mu_h$, $\lambda_2 = -(\pi+\sigma)$, $\lambda_3 = -(\mu_r)$, $\lambda_4 = -(\mu_h+\rho)$, $\lambda_5 = -[(\mu_h+\delta+\pi) - \frac{\rho X_h e_h}{\mu_h(\mu_h+\rho)}]$, $\lambda_6 = -(\mu_h+\delta+\pi)$, $\lambda_7 = -((\mu_r) - \frac{\varepsilon X_r e_r}{\mu_r(\mu_r+\varepsilon)})$ and from λ_5 and λ_7 one can easily establish that $R_{0h} < 1$ and $R_{0r} < 1$.

$\lambda_5 = -((\mu_h+\delta+\pi) - \frac{\rho X_h e_h}{\mu_h(\mu_h+\rho)})$, can be re-written as

$$\lambda_5 = -(\mu_h+\delta+\pi)(1 - \frac{\rho X_h e_h}{(\mu_h+\delta+\pi)(\rho+\mu_h)\mu_h}) \le -(1-R_{0h})(\mu_h+\delta+\pi) \tag{28}$$

Subsequently from (28) $R_{0h} < 1$

Also from $\lambda_7 = -((\mu_r) - \frac{\varepsilon X_r e_r}{\mu_r(\mu_r+\varepsilon)})$ one can write;

$$\lambda_7 = -(\mu_r)(1 - \frac{\varepsilon X_r e_r}{\mu_r^2(\mu_r+\varepsilon)}) \le -(\mu_r)(1-R_{0r}) \tag{29}$$

Which implies $R_{0r} < 1$

This implies that there would be fewer number of people infected with Lassa fever if proper control is implemented on the system, in other word the system will tends towards infection free equilibrium and this completes the proof of theorem 4.

Theorem 5: if $R_{0h}>1$ and $R_{0r}>1$ then the FOM (3) is asymptotically stable globally at (EE)

Proof.

According to Lasalle [35] method, we develop a function of lyapunov which is given below

$$L(S_r, E_r, I_r, S_h, E_h, I_h, R_h) \tag{30}$$

$$\begin{aligned} &= [S_h - S_h^*(\log\frac{S_h}{S_h^*})] + [E_h - E_h^*(\log\frac{E_h}{E_h^*})] + [I_h - I_h^*(\log\frac{I_h}{I_h^*})] + [R_h - R_h^*(\log\frac{R_h}{R_h^*})] + [S_r - S_r^*(\log\frac{S_r}{S_r^*})] \\ &+ [E_r - E_r^*(\log\frac{E_r}{E_r^*})] + [I_r - I_r^*(\log\frac{I_r}{I_r^*})] \end{aligned} \tag{31}$$

Differentiating (31) with respect t gives;

$$\begin{aligned} \frac{dL}{dt} &= (1-\frac{S_h^*}{S_h})\,{}_0^cD_t^\beta S_h(t) + (1-\frac{E_h^*}{E_h})\,{}_0^cD_t^\beta E_h(t) + (1-\frac{I_h^*}{I_h})\,{}_0^cD_t^\beta I_h(t) + (1-\frac{R_h^*}{R_h})\,{}_0^cD_t^\beta R_h(t) \\ &+ (1-\frac{S_r^*}{S_r})\,{}_0^cD_t^\beta S_r(t) + (1-\frac{E_r^*}{E_r})\,{}_0^cD_t^\beta E_r(t) + (1-\frac{I_r^*}{I_r})I_r(t) \end{aligned} \tag{32}$$

Replacing ${}_0^cD_t^\beta S_h(t)$, ${}_0^cD_t^\beta E_h(t)$, ${}_0^cD_t^\beta I_h(t)$, ${}_0^cD_t^\beta R_h(t)$, ${}_0^cD_t^\beta S_r(t)$, ${}_0^cD_t^\beta E_r(t)$, ${}_0^cD_t^\beta I_r(t)$

Equation (32) gives;

$$\begin{aligned} \frac{dL}{dt} &= (1-\frac{S_h^*}{S_h})[X_h - (\mu_h+\kappa_1)S_h + \sigma R_h + \gamma I_r] + (1-\frac{E_h^*}{E_h})[\kappa_1 S_h - (\rho+\mu_h)E_h] \\ &+ (1-\frac{I_h^*}{I_h})[\rho E_h - (\mu_h+\delta+\pi)I_h] + (1-\frac{R_h^*}{R_h})[\pi I_h - (\mu_h+\sigma)R_h] + (1-\frac{S_r^*}{S_r})[X_r - (K_2+\mu_r)S_r] \\ &+ (1-\frac{E_r^*}{E_r})[\kappa_2 S_r - (\mu_r+\varepsilon)E_r] + (1-\frac{I_m^*}{I_m})[\varepsilon E_r - (\mu_r+\gamma)I_r] \end{aligned} \tag{33}$$

According Lasalle principle [35]

For $E_r = E_r^*,\ S_r = S_r^*,\ I_r = I_r^*\ \ I_h = I_h^*\ \ S_h = S_h^*, E_h = E_h^*,\ R_h = R_h^*$,

We obtain $\frac{dL}{dt} = 0$

Hence the fractional order model (3) equipped with positive starting points,

$S_r = E_r = I_r = \ I_h = \ S_h = E_h = R_h \geq 0$ Are found in

$\Pi = \{S_r, E_r, I_r, S_h, E_h, I_h, R_h \in R_+^7; 0 \le N_m \le \frac{X_r}{\mu_r}; 0 \le N_h \le \frac{X_h}{\mu_h}\}$ Approaches the endemic equilibrium point P^*, it suffices that P^* is GAS. This completes the proof.

2.5 Numerical Solution for the (FOM) (3) in the Scope of Caputo Operator

We apply the operator of Caputo in fractional term to observe the dynamics of the FOM (3). The proposed nonlinear fractional-order system's numerical simulation is provided by the Adams type estimator-corrector approach [29, 31]. We consider the following Cauchy-type with respect to the order ß Caputo Operator.

$$ {}_0^c D_t^{\beta} \chi(t) = \Theta(t, \chi(t)), \qquad \chi^{(u)}(0) = \chi_0^u, \quad 0 < \beta \le 1, \quad 0 < t \le \alpha, $$
$$ \text{where } u = 0,1,2 \ldots n-1 \tag{34} $$

System (34) can be transformed into Voltera equation of the form

$$ \chi(t) = \sum_{\kappa=0}^{n-1} \chi_0^{(u)} \frac{t^u}{u!} + \frac{1}{\Gamma(\beta)} \int_{t_0}^{t} (t-\tau)^{\beta-1} \chi(\tau, \chi(\tau)) d\tau \tag{35} $$

This estimated corrector method is proposed with respect to algorithm of Adams bashforth-Moulten for us to achieve numerical values, taking the step size to be $b = \frac{\beta}{N}$, $t_i = ih$,

Where $i = 0.1.2 \ldots N \in Z^+$ and $X_i \approx X(t_i)$ which can be discretized equaling the corrector formular [29, 44-45]

$$ S_{h(n+1)} = \sum_{i=0}^{n-1} S_{0h}^{(i)} \frac{t_{n+1}^i}{i!} + \frac{b^{\beta}}{\Gamma(\alpha+2)} \sum_{i=0}^{n} (M_{i,n+1})[X_h - (\kappa_1 + \mu_h) S_{hi} + \sigma R_{hi} + \gamma I_{ri}] $$

$$ + \frac{b^{\beta}}{\Gamma(\beta+2)} \sum_{i=0}^{n} (M_{n+1,n+1})[X_h - (K_1 + \mu_h) S_{h(n+1)}^{PF} + \sigma R_{h(n+1)}^{PF} + \gamma I_{h(n+1)}^{PF}] $$

$$ E_{h(n+1)} = \sum_{i=0}^{n-1} E_{0h}^{(i)} \frac{t_{n+1}^i}{i!} + \frac{b^{\beta}}{\Gamma(\beta+2)} \sum_{i=0}^{n} (M_{i,n+1})[K_1 S_{hi} - (\rho + \mu_h) E_{hi}] $$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[K_1 S_{h(n+1)}^{PF}-(\rho+\mu_h)E_{h(n+1)}^{PF}]$$

$$I_{h(n+1)}=\sum_{i=0}^{n-1}I_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{i,n+1})[\rho E_{hi}-(\mu_h+\delta+\pi)I_{hi}]$$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[\rho E_{h(n+1)}^{PF}-(\mu_h+\delta+\pi)I_{h(n+1)}^{PF}]$$

$$R_{h(n+1)}=\sum_{i=0}^{n-1}R_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{i,n+1})[\pi I_{hi}-(\mu_h+\sigma)R_{hi}]$$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[\pi I_{h(n+1)}^{PF}-(\mu_h+\sigma)R_{h(n+1)}^{PF}]$$

$$S_{r(n+1)}=\sum_{i=0}^{n-1}S_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{i,n+1})[X_r-(K_2+\mu_r)S_{ri}]$$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[X_r-(\kappa_2+\mu_r)S_{r(n+1)}^{PF}]$$

$$E_{r(n+1)}=\sum_{i=0}^{n-1}E_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{i,n+1})[K_2 S_{ri}-(\mu_r+\varepsilon)E_{ri}]$$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[K_2 S_{r(n+1)}^{PF}-(\mu_r+\varepsilon)E_{r(n+1)}^{PF}]$$

$$I_{r(n+1)}=\sum_{i=0}^{n-1}I_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{i,n+1})[\varepsilon E_{ri}-(\mu_r+\gamma)I_{ri}]$$

$$+\frac{b^{\beta}}{\Gamma(\beta+2)}\sum_{i=0}^{n}(M_{n+1,n+1})[\varepsilon E_{r(n+1)}^{PF}-(\mu_r+\gamma)I_{r(n+1)}^{PF}]$$

Where $M_{i,n+1}=\begin{cases}n^{\beta+1}-(n-\beta)(n+1)^{\beta}\\(n-i+2)^{\beta+1}+(n-i)^{\beta+1}\\1\end{cases}$

The above constrains hold if $i=0$, $1\le i\le n$, $i=(n+1)$ respectively

The next stage is to create the concurrence predictor formula X_{n+1}^{PF}, the predictor formula can be given as;

$$S_{h(n+1)}^{PF}=\sum_{i=0}^{n-1}S_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[X_h-(\kappa_1+\mu_h)S_{hi}+\sigma R_{hi}+\gamma I_{ri}]$$

$$E_{h(n+1)}^{PF}=\sum_{i=0}^{n-1}E_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[K_1 S_{hi}-(\rho+\mu_h)E_{hi}]$$

$$I_{h(n+1)}^{PF}=\sum_{i=0}^{n-1}I_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[\rho E_{hi}-(\mu_h+\delta+\pi)I_{hi}]$$

$$R_{h(n+1)}^{PF}=\sum_{i=0}^{n-1}R_{0h}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[\pi I_{hi}-(\mu_h+\sigma)R_{hi}]$$

$$S_{r(n+1)}^{PF}=\sum_{i=0}^{n-1}S_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[X_r-(K_2+\mu_m)S_{ri}]$$

$$E_{r(n+1)}^{PF}=\sum_{i=0}^{n-1}E_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\alpha+1)}\sum_{i=0}^{n}(L_{i,n+1})[K_2S_{ri}-(\mu_r+\varepsilon)E_{ri}]$$

$$I_{r(n+1)}^{PF}=\sum_{i=0}^{n-1}I_{0r}^{(i)}\frac{t_{n+1}^{i}}{i!}+\frac{b^{\beta}}{\Gamma(\beta+1)}\sum_{i=0}^{n}(L_{i,n+1})[\varepsilon E_{ri}-(\mu_r+\gamma)I_{ri}]$$

Where $L_{i,n+1}=(n+1-i)^{\beta}-(n-i)^{\beta}$

2.6 Congenital Trace and Memory tracking

To capture the dynamics of Lassa hemorrhagic fever (LHF) model (3) we utilize the defined Caputo operator in fractional order model (3) where; *0<β≤1*, let *X(t)* represent the fractional order [29, 45]

$${}_0^cD_t^{\beta}\chi(t)=\Psi(\chi(t),t) \tag{36}$$

Using the approach of L1 scheme [38] the numerical estimate of the functional derivative of *X t)* is

$${}_0^cD_t^{\beta}\chi(t)\approx\frac{(dt)^{-\beta}}{\Gamma(2-\beta)}\left[\sum_{\kappa=0}^{J-1}[\chi(t_{\kappa+1})-\chi(t_{\kappa})][(J-\kappa)^{1-\beta}-(J-1-\kappa)^{1-\beta}\right] \tag{37}$$

L1 scheme is one of the most accurate approach used in partitioning fractional derivative due to its memory term and rate of convergence. Using (28) and (29)

$$\chi(t_J)\approx{}_0^cD_t^{\beta}\Gamma(2-\beta)F(\chi(t),t)+\chi(t_J-1)-\left[\sum_{\kappa=0}^{J-2}[\chi(t_{\kappa+1})-\chi(t_{\kappa})][(J-\kappa)^{1-\beta}-(J-1-\kappa)^{1-\beta}\right] \tag{38}$$

The fractional derivative solution could be term the variance between the memory tracking and term of Markov, where the Markov term is fitted by a function of gamma

$\chi(t_J) \approx D_t^{\beta}\Gamma(2-\beta)F(\chi(t),t) + \chi(t_J - 1)$ = Term of Markorv

$\left[\sum_{\kappa=0}^{J-2}[\chi(t_{\kappa+1}) - \chi(t_\kappa)][(J-\kappa)^{1-\beta} - (J-1-\kappa)^{1-\beta}\right]$ = Memory Tracking

For $\beta < 1$, all past analysis of disease is taken into account for memory tracking and the memory tracking is solely based on time.

3 RESULTS and DISCUSSION

Numerical simulation was carried out using MATLAB software to check the impact of the fractional order ß and saturation (treatment) on the fractional order model (3), the Adams Bashforth Predictor-corrector technique has been inserted into the numerical scheme.

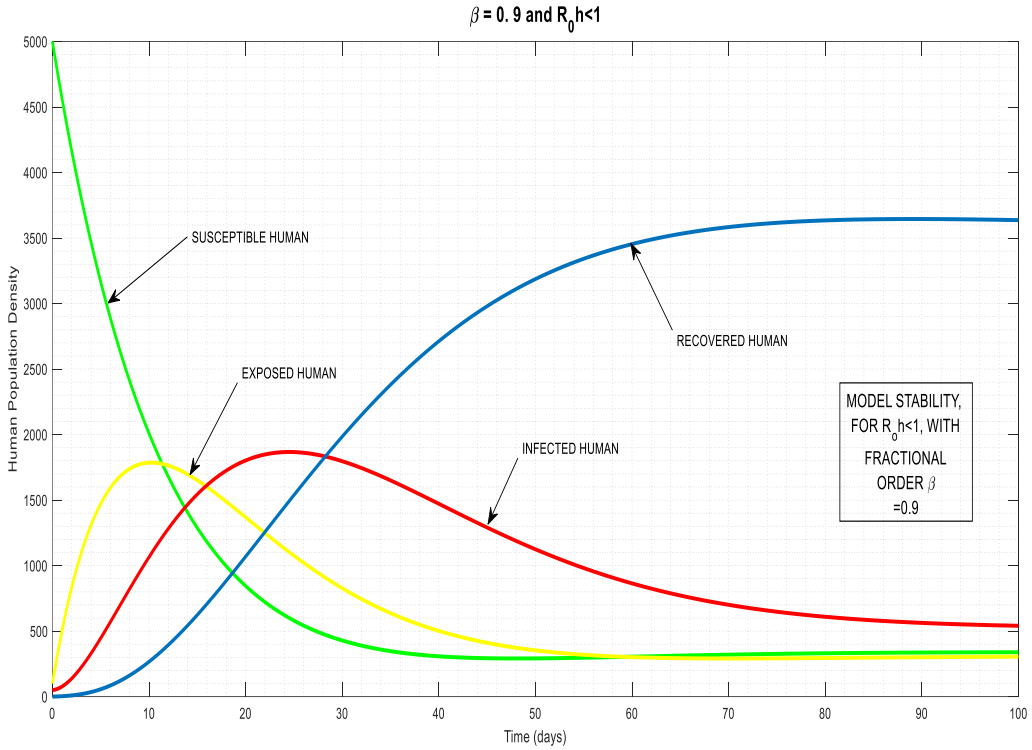


***Figure 1**. Model stability solution of human population density with fractional order ß=0.9 and for $R_{0h} < 1$ demonstrating the well-Posed model.*

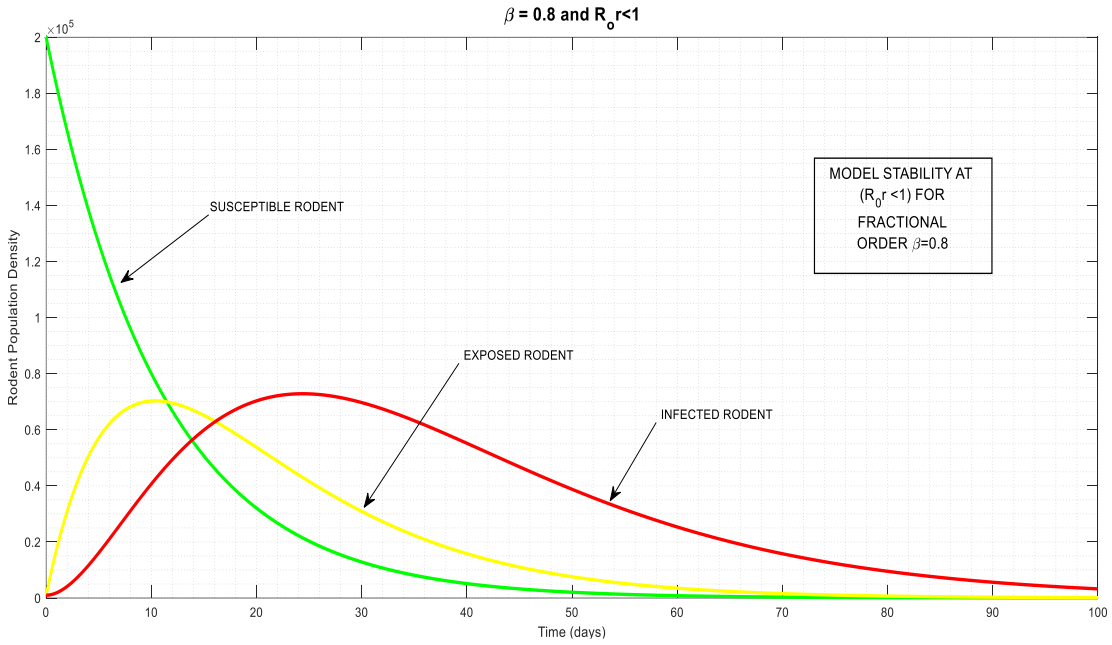


***Figure 2**. Model stability solution of rodent population density with fractional order Ɓ=0.8 and for $R_{0r} < 1$ demonstrating the well-Posed model.*

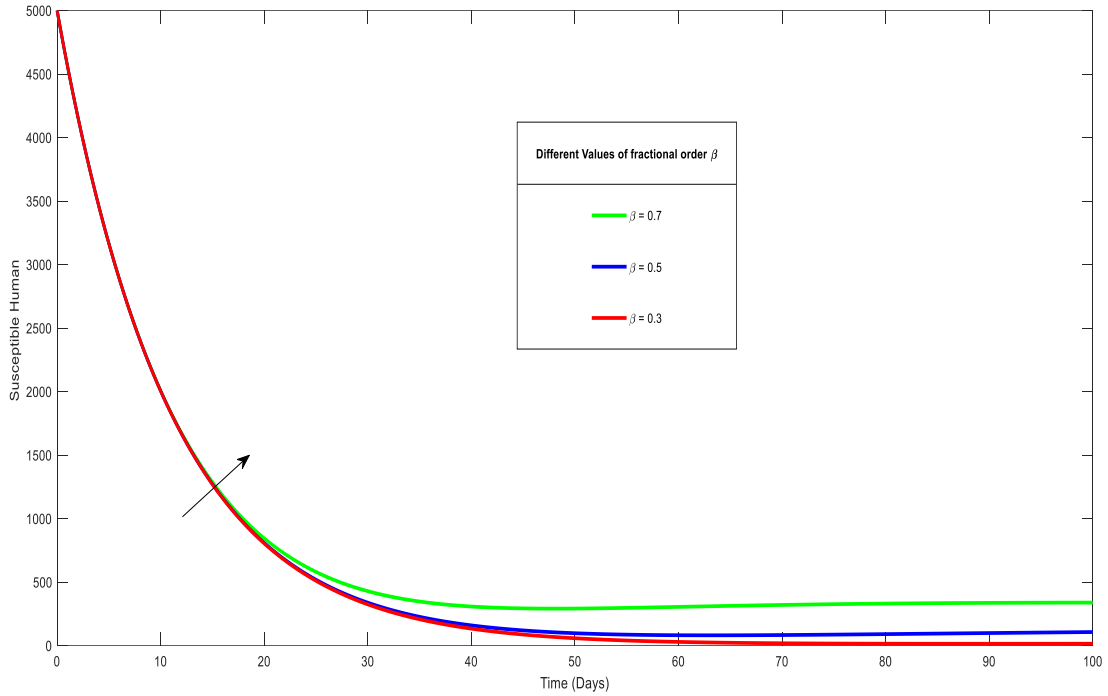


***Figure 3**. Solution trajectory of fractional order Ɓ = 0.3, 0.5, 0.7 on the human susceptible, demonstrating increment as the fractional order increases.*

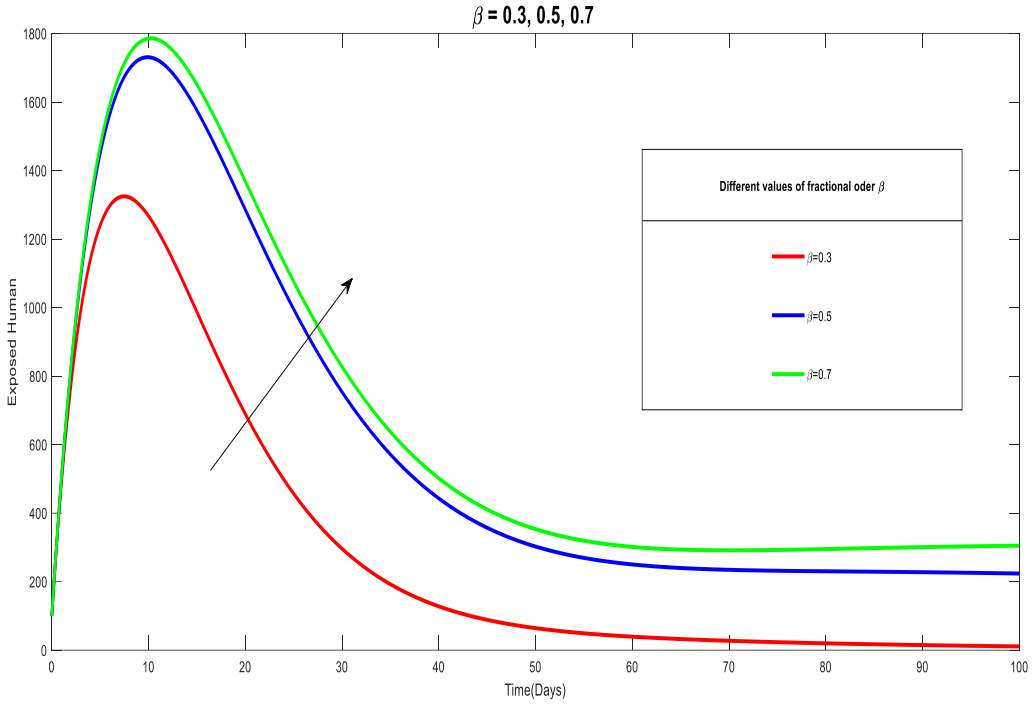


Figure 4. *Solution trajectory of fractional order Β on the human exposed, demonstrating increment as the fractional order increases.*

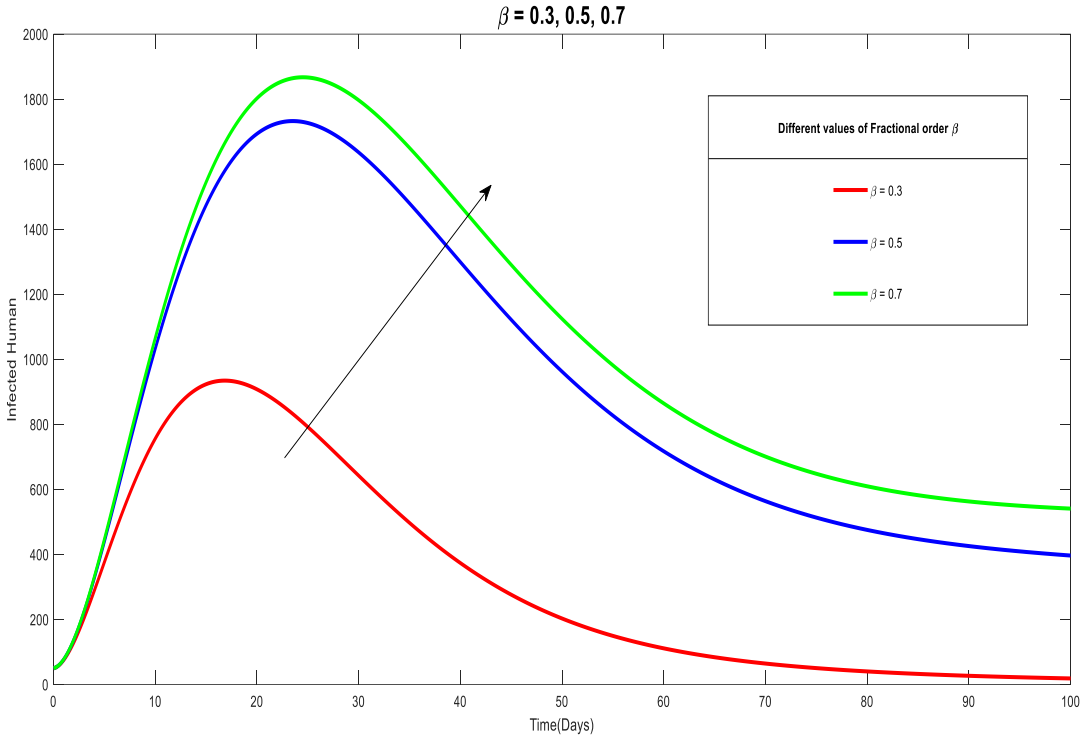


Figure 5. *Solution trajectory of fractional order Β=0.6 on the human infected, demonstrating increment as the fractional order increases.*

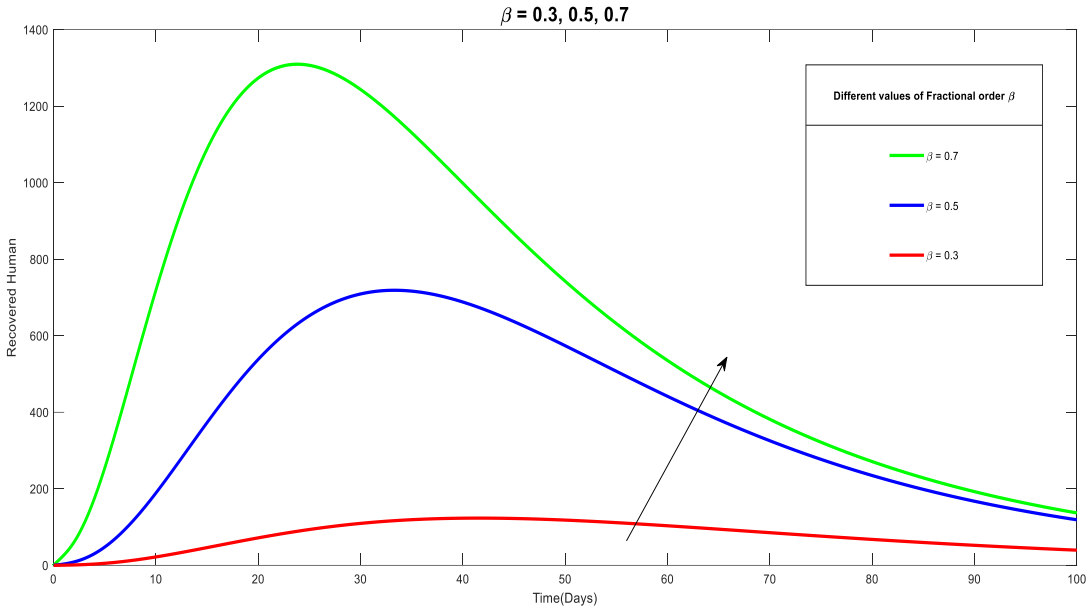


***Figure 6**. Solution trajectory of fractional order β on the human recovered, demonstrating increment as the fractional order increases.*

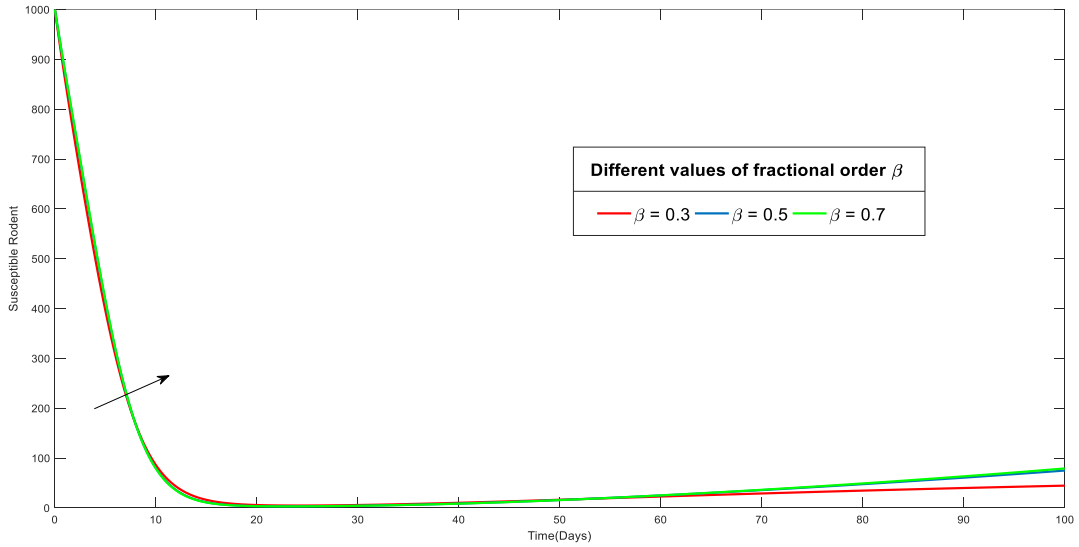


***Figure 7**. Dynamical solution of fractional order β on susceptible rodent class.*

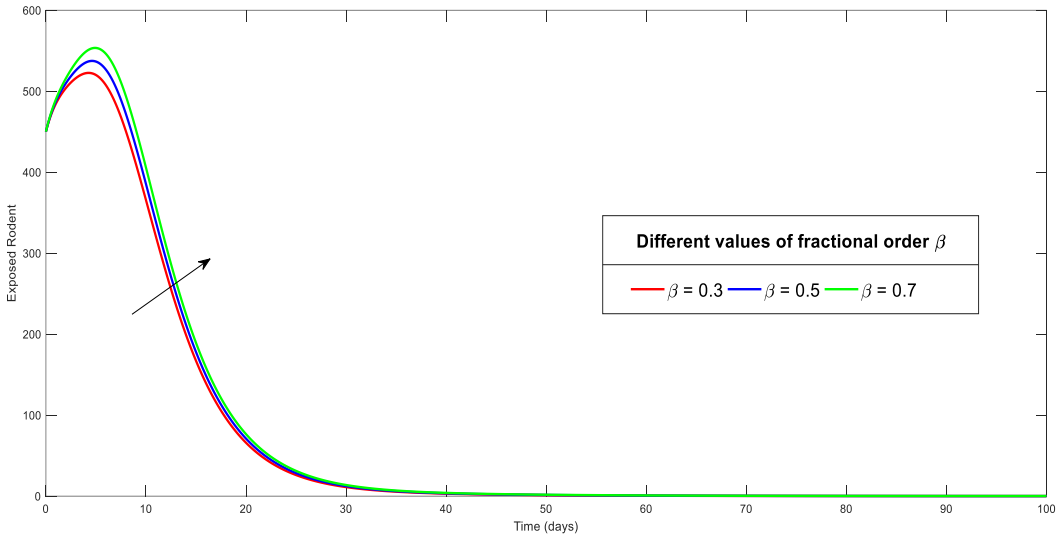


***Figure 8**. Dynamical solution of fractional order B on exposed rodent class.*

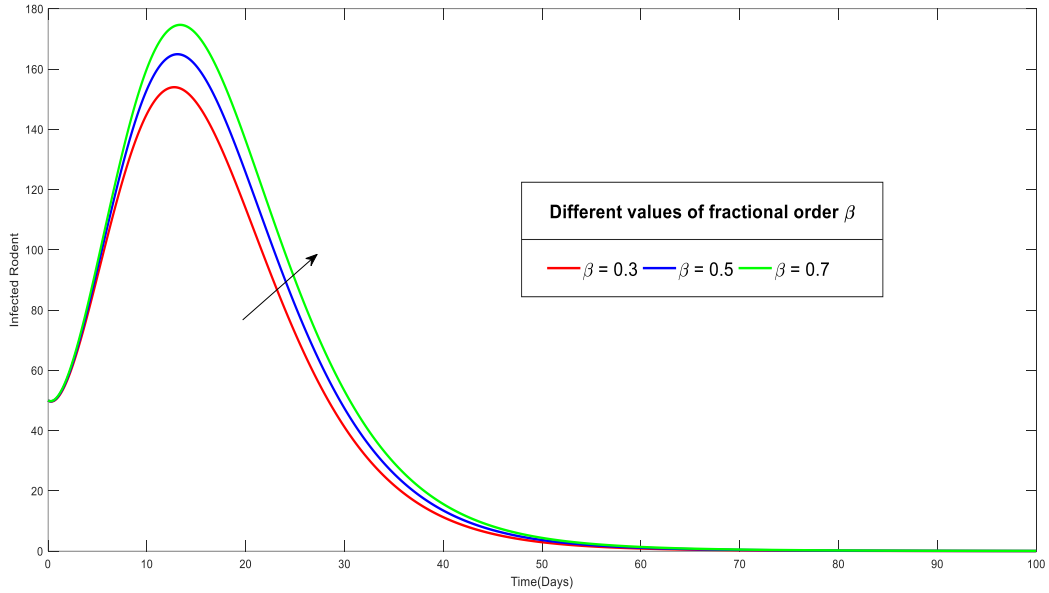


***Figure 9**. Dynamical solution of fractional order B on the infected rodent.*

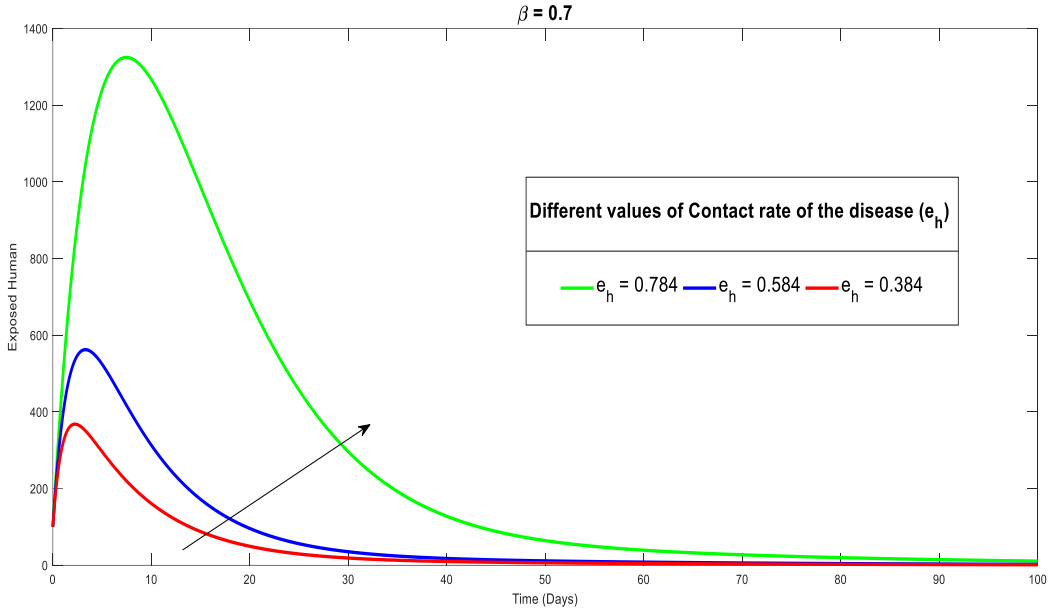


Figure 10. *Dynamical behavior of exposed human while varying the effective contact rate (e_h) of the disease with fractional order Ɓ =0.7.*

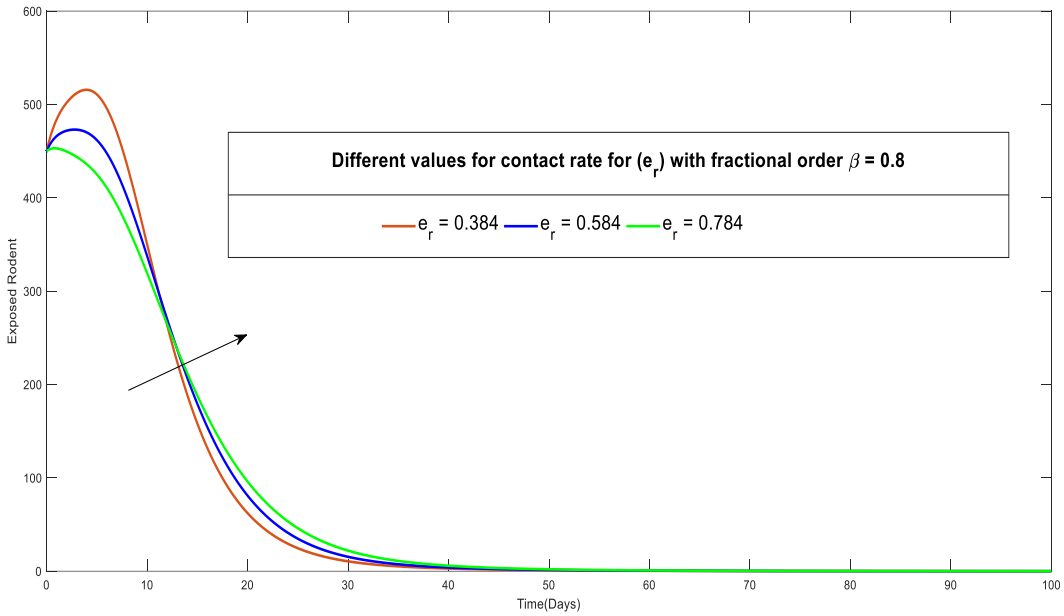


Figure 11. *Dynamical behavior of different values of disease contact rate for rodent class (e_r) with fractional order Ɓ = 0.8.*

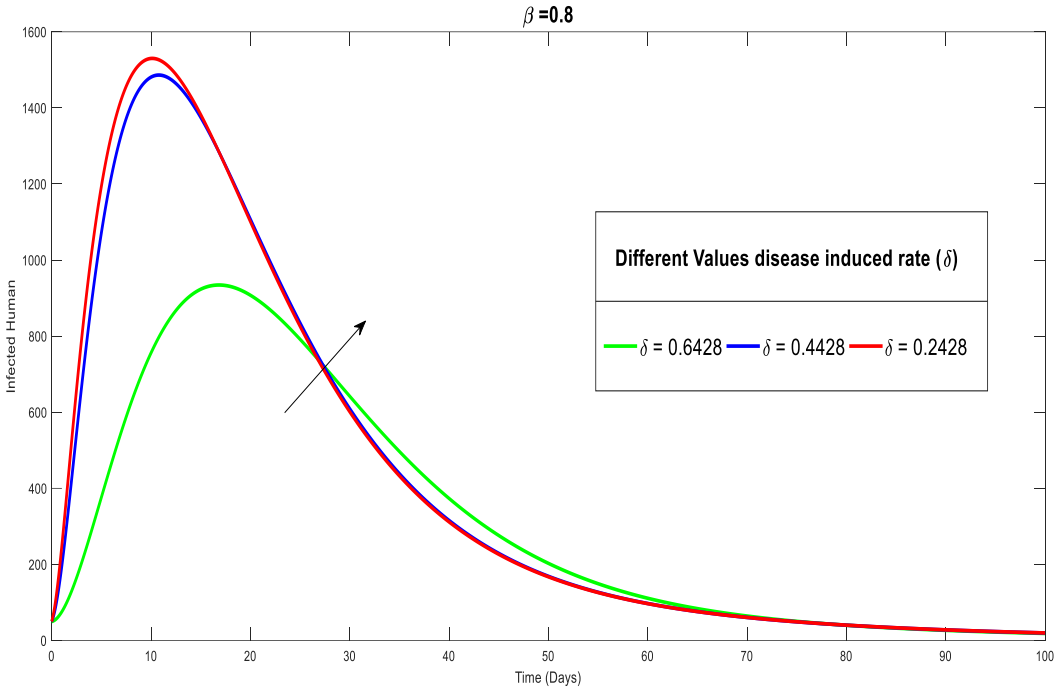


Figure 12. *Dynamical behavior of infected human with different values of disease induced rate (δ) with fractional order β =0.8.*

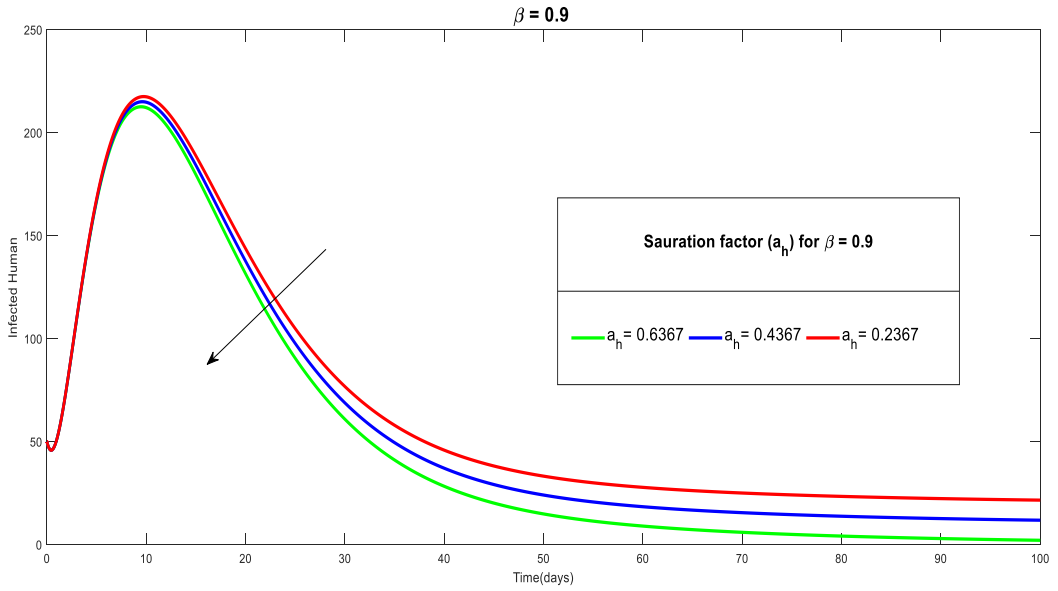


Figure 13. *Dynamical solution of infected human for fractional order β = 0.9 with different values of saturation factor* (a_h).

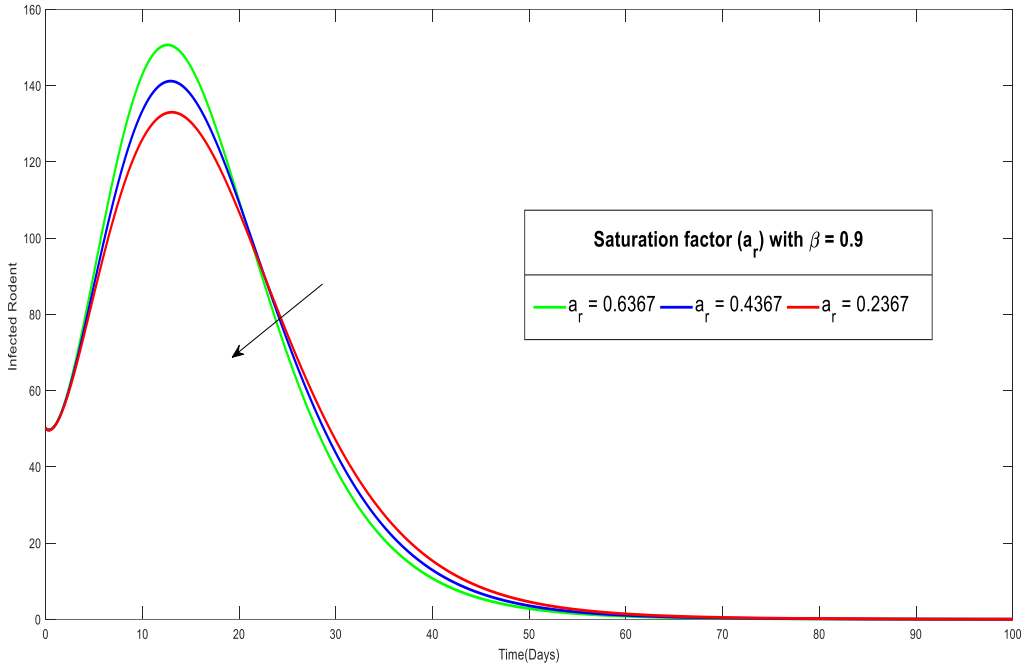


***Figure 14**. Dynamical behavior of different values of saturation factor with fractional order ß =0.9.*

The stability and Mathematical well-posed of the FOM (3) was illustrated in Figure: 2 and Figure: 3 for fraction order $\beta = 0.9$, $\beta = 0.8$ and $R_{0h} < 1$ and $R_{0r} < 1$ respectively, to investigate the dynamical behavior of the proposed model, where; $X_h = 0.09,\ \kappa_1 = 0.09196,\ \sigma = 0.076,\ \rho = 0.010,\ \pi = 0.009,\ X_r = 0.18,\ \kappa_2 = 0.09198,$ $\varepsilon = 0.03495,\ \delta = 0.001865,\ \mu_h = 0.001127,\ \mu_r = 0.097,\ \gamma_r = 0.037,$ and $\kappa_2 = 0.09199,$ are all fitted values.

With the fitted values given, different values of fractional order β was incorporated in each class to check the behavior of the fractional model (3), Figure: 4, 5, 6, 7 which is the human population density increase as the fractional order ß rises over course of 100 days.

Similarly Figure: 8,9,10 also show an increment as the fractional order ß increase, depicting the effect of the fractional order on the population density of rodent.

In Figure: 11, and 12, the effect of disease contact rate $(e_h) and (e_r)$ was incorporated in the exposed human and exposed rodent respectively for $(\beta = 0.7, 0.8)$ to check the behavioral solution, which also increases as the effective contact rate got higher, its attendant effect was demonstrated in Figure: 13 which

is the death caused by the infection rate (δ_h) for infectious human, if proper control plan is not put in place it might tends to endemic point.

In Figure: 14, the saturation factor (a_h) for the fractional order ß=0.9 was incorporated into the infectious human to check the dynamics which depicts a decrease in this set of individuals translating to flattening out the disease (saturation in this context means treatment).

Figure: 15 saturation factor (a_r) was inserted in the infectious rodent where the fractional order $\beta = 0.9$, demonstrating a sharp decrease in this class of rodent, saturation here could be (use of pesticide, fumigation, proper hygiene and likes).

4 CONCLUSION

Using a modified fractional order within the scope of Louiville-Caputo, equipped fractional order ß, we have inquired a Lassa hemorrhagic fever (LHF). By using a fixed-point approach, we've shown the uniqueness, existence of the fractional model. Furthermore, we established the boundedness and positivity of the fractional order model within feasible regions. Additionally, the points equilibrium for endemic and disease free status were verified, and the next-generation matrix approach was used to establish the rate at which the infection multiply within the proposed model (R_0) which was found to be less than (1) via the stability analysis using the Jacobi approach. The stability analysis of the endemic state was verified using Lyapunov technique for ($R_0 > 1$). Via numerical scheme, using the Adams bashforth estimator-corrector approach the numerical solution was derived. The findings explain the effect of different parameters on the model. Significantly, it is found that the present model's integration of memory impacts through Caputo (FOD) affects how quickly solution paths approach to steady state solution. These results add to the body of data supporting the idea that fractional operators give more insightful, dependable, and insightful understanding of the (LHF) fractional model with memory trace.

Conflict of Interest

There is no conflict of interest between the authors.

Authors Contributions

Adedapo Chris LOYINMI: Conceptualization, Formal analysis, methodology, review and supervision.

Alani Lateef IJAOLA: Conceptualization, methodology, data analysis writing of original draft.

Emmanuel Oyeemi Adeleke: Writing, methodology and data analysis.

Olwofeso Oluwatoyin: Resources, writing and editing.

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Statement of Research and Publication ethics.

The study is compiled within research and publications ethics.

Artificial Intelligence (AI) Contribution Statement

This manuscript was entirely written, edited, analyzed, and prepared without the assistance of any artificial intelligence (AI) tools. All content, including text, data analysis, and figures, was solely generated by the authors.

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